Phenomenologism vs fundamentalism: The case of superconductivity

Towfic L. Shomar

This article argues that phenomenological treatment of physical problems is more powerful than fundamental treatment. Developments in the field of superconductivity present us with a clear example of such superiority. The BCS (Bardeen, Cooper and Schrieffer) was accepted as the fundamental theory of superconductivity for a long time. Nevertheless, Landau and Ginzburg phenomenological model has so far proven to be a more fruitful theoretical representation to understand and to predict the features of superconductivity and superconductive materials.

Keywords: BCS theory, fundamentalism, Landau and Ginzberg, phenomenalism, superconductivity.

TWO distinct types of theoretical work can be found in physics. The main difference between them is the way in which they are built. One starts from abstract entities to accommodate experimental observation and the other proceeds in the opposite direction[1].

The first is expressed in 'fundamental' theories. I claim that fundamental theories are merely tools for constructing both models and other fundamental theories. This article will show that previous fundamental theories play the role of tools in constructing new fundamental theories (note 1). They do not express truth about nature. They are theories that:

(1) Give a coherent story about the studied phenomena.
(2) Can explain existing empirical findings in the field and can predict new aspects of the phenomena under study that can be tested empirically.
(3) Are consistent with previous theories.
(4) Are either derivable from previous accepted fundamental theories; or at least correspond at a certain limit to these previous theories; or must be able, at least, to provide a mathematical basis for a net of theoretical models that can provide good explanations to the properties of the field of study.
(5) Are in principle able to be unified with each other.

Hence, the basis for fundamentalism is mathematical in structure. How a fundamental theory is constructed is given later in the article.

The second type of theoretical representation is that which does not have a deductive relation to previous established fundamental theories. This kind of theoretical work is closely concerned with experimental work and takes as a point of departure a descriptive account of experimental activity. Models constructed in this way are phenomenological models and they have the following features:

(1) Are a special kind of theoretical representation that departs from the phenomenological level.
(2) Describe the phenomena, including all the factors that would allow them to exhibit themselves, with whatever experimental boundary conditions deemed necessary.
(3) Represent natural phenomena in as far as the model is able to mediate between the experimental and the theoretical. They do so by presenting a story that can relate the mathematical parameters of the models to the properties known about the phenomenon.

A simple description of what physicists do will illustrate what a phenomenological model is. If a new phenomenon is discovered (or built), physicists try to understand it. A group of usual procedures is applied. The first of these is to know the conditions in which the phenomenon occurs. In accordance with these conditions, physicists suggest a set of experiments that would help in generating new data. At this point our knowledge of the great variety of theories available in physics will be important. Physicists will search within all possible theoretical schemes to find a mathematical structure that can represent the data. They do this by 'plotting the data'; they then begin thinking about which of the known mathematical forms used in physics can mimic the plotted data. These mathematical forms need not be related to the field of the phenomenon.

Now, it would not be enough to find a mathematical form that expresses the data pattern formally. The 'physical intuition' of the physicists would lead them to relate the

---

Towfic L. Shomar is in the Department of Human and Social Sciences, Philadelphia University, Jordan and the Centre for the Philosophy of Human and Social Science, London School of Economics, London.
e-mail: T.L.Shomar@lse.ac.uk

parameters of the mathematical form to the properties of the phenomena. The established relations between the parameters and the properties need, in order to adhere to coherency, to be consistent with our previous established pictures of the related fields. In order to do so the model would need a descriptive body that would present a coherent picture, consistent with our previously established models.

At this stage we have only a scattered descriptive measures, with a set of mathematical forms and a set of possible connections with established models in physics. Physicists would start to present an overall model that will account for all the known features of the studied phenomenon. If the outcome scenario succeeds in relating the properties to the parameters, then the mathematical form together with the descriptive body and the story would constitute a phenomenological model.

However, the ‘physical intuition’ does not mean relating the data to an existing theory already agreed to cover the phenomenon, but possibly changing a tool from an understood theory, in another domain, to adapt it to what is studied. Hence, the ‘physical intuition’ plays a role in presenting a story which can relate the data and the ontological world of the phenomenon with the expected mathematical structure. Construction of a phenomenological model is presented later in the article, showing how the phenomenological aspects and different tools from different fundamental theories help in this construction (note 2).

It is important to mention that knowing the mathematical structure, which might be used to illustrate the data, will not provide enough information to build a model. I do not accept, for example, that a phenomenological model is merely a mathematical structure (note 3). At the same time, it is not naked data, there is more to it. The story which provides the basis for how to deal with the mathematics in relation with the phenomenon, is as crucial as finding a mathematical structure. Hence a phenomenological model in physics consists of a mathematical structure plus a descriptive level, depending on the human activity, that is driven by our experience and by the empirical data themselves and has two levels: the objective description of the experimental set-up or the environmental set-up and a kind of explanation that provides a deeper understanding to the phenomenon. Hence, a phenomenological model is a type of theoretical explanation that departs from the description of the environmental set-up of the phenomenon to give a structural account of its relations. However, the theoretical explanation that the phenomenological model provides is not as abstract as the fundamental theory explanation.

It should be noted here that I am changing the term experimental set-up with environmental set-up because phenomenological models try to capture nature within its boundaries and not to impose purified boundaries such as those imposed by the experimental set-up. That is to say, the experimental set-up usually takes the natural phenomenon from its natural setting and tries to eliminate all possible distractive elements before conducting the experiment. Phenomenological models, because of the way they are built (i.e. from a bottom-up approach), try to capture as much of the natural environment as possible, regardless of the ability to understand all the used factors. As long as the model is able to represent the overall features of the phenomenon, it would be accepted as a phenomenological model.

Although the mathematical form of the phenomenological model is similar in nature to fundamental theories, it is usually associated with some parameters that cannot be specified theoretically, but are found through experimentation and fed into the model at certain points during the building process. By virtue of such parameters, the phenomenological model might be counted as low-level theorization. Hence, this would be a good reason to think of the explanation provided by the phenomenological model as a whole, as being less abstract.

Therefore, the main differences between a fundamental theory and a phenomenological model are:

(1) Fundamental theories are constructed using a top-down approach, while phenomenological models are constructed using a bottom-up approach.
(2) Phenomenological models tend to be more flexible.
(3) The story presented by the phenomenological model is not as abstract as that presented in the fundamental theory.
(4) The phenomenological models need not tell us why certain phenomena behave the way they do; it is sufficient that they give good predictions about any phenomenon.

Furthermore, with the help of different tools from the so called ‘fundamental’ theories, physicists can construct new phenomenological models to account for any new observed properties. It renders them with higher flexibility than fundamental theories.

This entails a deeper thesis: fundamental theories need not play a representative role. They only serve as tools in developing more theories that are fundamental and for constructing phenomenological models; the latter, from a realist point of view, can be accepted as representative of nature.

Before indulging in the Bardeen, Cooper and Schrieffer (BCS) vs Landau and Ginzburg (LG) discussion, it might be useful to look at a simpler example. Let us consider Newton’s second law. If we are thinking of the Newtonian framework, then we ought to interpret the law in accordance with moving bodies in a three-dimensional world in an infinite space and infinite time. Also, in the fundamental level of the law we present it in the most general possible form:

$$\sum F = a \sum M, \tag{1}$$

where F is the force, M the mass and a the acceleration.

However, when we want to apply such a general law on a particular setting, we start to add certain concepts and elements to account for the specific feature of such a setting. Let us consider, for example, the case of a box sliding on a rough surface with an angle θ with the horizon. In such a case the exact model that accounts for this particular case would be:

$$F = mg \cos \theta - R = ma, \tag{2}$$

where g is the gravitational constant and R the resistant. In this specific case, we are dealing with a specific model that can be interpreted as the outcome of a series of observations of moving bodies over slopes with known friction. Although it depends on our previous knowledge of trigonometry and a generalized model, Newton's second law, it might also be inductively inferred from direct observations and mathematical knowledge. This model is still true, whether we accept the Newtonian absolute space and absolute time, or the relativistic space–time curvature. Hence, such a model with its connotations and associated story would be considered as a low-level theoretical representation and would be counted as a phenomenological model. Here in such a case, we are differentiating between the high-level generalization and low-level representative model. The first expresses a general law that can be generic of any force in certain space, while the latter expresses a specific case with specific features.

The more complex the theoretical representation, clearer is the distinction between fundamental theories and phenomenological models. In case of Maxwell's equations, for example, some of the terms in the theoretical representation of the fundamental theory do not have a direct counterpart in nature, such as the electric displacement D, which has no clear physical meaning[5].

The article studies the contrast between a fundamental theory, say the BCS theory of superconductivity and the LG-phenomenological model of superconductivity, a model accepted now as the best representative of superconductivity. It then discusses the similarities and differences between these two theoretical works, and later argues that new discoveries bring into question the very idea of having a fundamental theory for all kinds of superconductivity.

Superconductivity

Superconductivity was first discovered by the Dutch physicist Onnes in 1911. He detected that metals when cooled to a very low temperature, inside liquid helium (under 4 K), exhibit a strange phenomenon: the total disappearance of resistance under a critical transition temperature T_c. Later, in 1933, Meissner and Ochsenfeld discovered that the magnetic field is expelled inside the superconductor under a certain transition magnetic field $\boldsymbol{H}_c$ (the Meissner effect)[6].

The discovered experimental constrains shaped the theoretical understanding of superconductivity from the beginning. In 1935, London and London[7] (note 4) suggested the first theoretical explanation of superconductivity. Using their knowledge from diamagnetism, they constructed a phenomenological model expressed by the London equations:

$$\operatorname{curl} \Lambda \boldsymbol{j}_s + \boldsymbol{H} = 0, \quad \frac{\partial}{\partial t}(\Lambda \boldsymbol{j}_s) - \boldsymbol{E} = 0, \tag{3}$$

where $\boldsymbol{j}_s$ is the superconducting current density, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{E}$ the electrical field, c the speed of light and Λ is an experimental constant $= m/ne^2$, where m is the mass of the electron, n the number of electrons and e the charge of the electron.

Later, a number of experimental activities tested the correctness of these equations. The results show that London's equations could give good predictions for the superconducting current if the following conditions are satisfied:

(1) $\boldsymbol{H} << \boldsymbol{H}_c$, otherwise at the boundary zone where $\boldsymbol{H}$ is near the critical magnetic field, the model failed to predict what happens experimentally.
(2) The superconducting electron density must be constant.
(3) The penetration depth is less than the thickness of the sample.

The difficulties facing London equations are as follows. First, when the temperature $T \to T_c$, the penetration depth becomes larger and the thickness of the 'walls' separating the normal and the superconducting states also become larger.

The second difficulty is related to the change in the free energy between the normal and superconducting states. As pointed out by Ginzburg[9], 'if we restrict ourselves to the case of a steady field, then (London's equations), together with Maxwell's equations, are sufficient for determining the density $\boldsymbol{j}_s$ of the superconducting current and the field $\boldsymbol{H}$ in the superconductor'. However, if we want to have a broader understanding of the change from the normal to the superconducting state, the London approach is not sufficient.

Any new theory or model should also aim at explaining additional empirical facts:

(1) The transition temperature is proportional to the isotopic mass M of the metal nuclei (the isotope effect)[10].

(2) The phase transition at the critical temperature would transfer the electrons from a disordered bunch to an ordered one.
(3) The thermal vibration of the atoms is the principal cause of electrical resistance in metals at ordinary temperature[10].
(4) The energy gap.

Two schemes were developed to overcome the difficulties which the London model faces: the LG phenomenological model and the BCS fundamental theory.

LG model

In 1950, LG developed 'an extension of the London phenomenological' model 'to take into account a space variation of the order parameter'[11]. They suggested the following equations:

$$\alpha\psi + \beta\,|\psi|^2\,\psi + \frac{1}{2m}\left(-i\hbar\nabla - \frac{2e\boldsymbol{A}}{c}\right)^2 \psi = 0,$$

$$\boldsymbol{j} = -\frac{ei\hbar}{2m}(\psi^*\,\nabla\psi - \psi\nabla\psi^*) - \frac{e^2}{mc}\psi^*\,\psi\boldsymbol{A}, \qquad (4)$$

where α, β are experimental constants, ψ a pseudo wave function and $\boldsymbol{A}$ the local vector potential.

How did they arrive at their conclusions? Seeing how they did so will help make clear why such a model is called 'phenomenological'.

When London and London suggested their solution for the Meissner effect, they ended their paper by stating that a more general solution for superconductivity might be inspired by studying Gordon's formulae for electric current:

$$\boldsymbol{J} = \frac{he}{4\pi im}(\psi\nabla\psi^* - \psi^*\nabla\psi) - \frac{e^2}{mc}\psi\psi^*\boldsymbol{A},$$

$$\rho = \frac{he}{4\pi imc^2}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right) - \frac{e^2}{mc^2}\psi\psi^*\phi. \qquad (5)$$

This is in effect the mathematical form that London and London expected would represent the experimental data of superconductivity. However, London and London failed to present a story that could identify the parameters in the equation with properties of superconductivity. So, LG started from this remark in addition to the experimental evidence. The idea was to try to find tools from existing theories and to borrow forms that might end up with a similar equation to that of Gordon's, but which can be directly related to the field of superconductivity.

What kind of information can be provided by the experimental and phenomenological facts? And what kind of story can be told? At that time it was an experimental fact that superconductivity exhibited some type of thermal fluctuation. The thermal vibration is known to be the cause of electrical resistivity. Hence the no-resistivity region should be related to what could happen for such thermal vibration.

Landau had been working on a theory for the phase transition in solid-state physics. He thought that it could be of aid in their derivation. Experimentally, it was expected that transition at the critical temperature is one from disorder to order. However, there is more than one way to represent the shift from disorder to order. Nevertheless, the experimental evidence helped Landau suggest that the transition from the normal to the superconducting state is a second-order transition (as in the transition from ferromagnetic to paramagnetic), where we can neglect the higher terms. This means that the thermal vibration of the electrons that cause electrical resistivity will be ordered under the transition from the normal to the superconducting state. No thermal vibration will occur anymore, which implies that no resistivity will occur as well.

London and London suggested that the flow of electrons through the sample can be considered as two fluids: superconducting and normal. Hence, it was normal to assume that the superconducting state would be presented thermodynamically. Then to relate the transition at the critical temperature to thermodynamics, LG offered a 'guess' for the free energy:

$$\int F(\Delta; T, \boldsymbol{E})\mathrm{d}^3 r,$$

where Δ is an order parameter which is function of the distance r (Schrieffer[12]). Expanding this function in terms of power series in Δ^2 we find that

$$F = -\boldsymbol{E}\Delta + g_0 + \frac{1}{2}g_2\Delta^2 + \frac{1}{4}g_4\Delta^4 + \cdots.$$

The standard text by Tilley and Tilley[13] describes this saying: 'Landau's general theory of second-order phase transitions is based on the idea that a phase transition could be characterized by some kind of order parameter, and a simple postulated form for the dependence of the free energy on the order parameter'.

Consider the non-vanishing terms. Because it is a second-order phase transition, the term g_4 is positive and the higher-order terms can be neglected (note 5). Think about the transition from the normal to the superconducting states as one from disorder to order, and take the normal free energy to have the same form as in thermodynamics, which is:

$$F = U - T\Sigma,$$

where U is the initial energy.

In the case of a variable magnetic field, the relation between the free energy and the order parameter is:

$$F = F_n + a\Delta^2 + \frac{b}{2}\Delta^4 + C\left|\left(-i\nabla - \frac{2e\boldsymbol{A}}{\hbar c}\right)\Delta\right|^2 + \frac{\boldsymbol{B}^2}{8\pi}. \quad (6)$$

And in order to keep the invariance, there is an added vector potential ***A***, which is related to the field ***B*** ($\boldsymbol{B} = \mathrm{curl}\,\boldsymbol{A}$).

Tilley and Tilley[13] continue: the next 'crucial insight in LG was that for a superconductor the order parameter **must** be identified with the macroscopic wave function ψ'. As I said, LG were inspired by Gordon's equations. To correspond their equation to that of Gordon, they ought to identify a parameter in their equation with the wave function. The only parameter which can be thought of as conveying the same properties as ψ is the order parameter. So, in this sense it was a must for LG to identify the order parameter with the macroscopic wave function ψ.

Then by setting the correct values of the coefficients, depending on the experimental results we get:

$$F = F_n + \alpha\psi^2 + \frac{\beta}{2}\psi^4 + \frac{1}{2m}\left|\left(-i\hbar\nabla - \frac{2e\boldsymbol{A}}{c}\right)\psi\right|^2 + \frac{\boldsymbol{B}^2}{8\pi}. \quad (7)$$

It may seem that F has just been modified from eq. (6) to eq. (7), but this is far from correct. Two features of the construction illustrate my thesis about the use of theories as a tool for constructing phenomenological models: first, eq. (6) itself had been constructed using tools from different theoretical models which are not connected to superconductivity. These tools are: the phase transition, the assumption that the free energy is a function of the order parameter and the Gordon's equations. Second, eq. (7) is not derived from quantum mechanics, rather a non-quantum mechanical eq. (6) is reformed in eq. (7). This illustrates a theoretical influence that functions merely as a way to express the experimental results. Physicists see that 'this construction of eq. (7), independent of any detailed theory of the superconducting state, represented a *tour de force* of physical intuition'[14].

Now from the modification of the free energy in eq. (7), LG set $\partial F = 0$, to obtain the following equations:

$$\alpha\psi + \beta\,|\,\psi\,|^2\,\psi + \frac{1}{2m}\left(-i\hbar\nabla - \frac{2e\boldsymbol{A}}{c}\right)^2\psi = 0,$$

$$\boldsymbol{j} = -\frac{e^* i\hbar}{2m^*}\,(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e^{*2}}{m^* c}\psi^*\psi\boldsymbol{A}. \quad (8)$$

As we can see, these equations have the same form as Gordon's equations even if there is no direct relation in a deductive or physical sense (note 6). Gordon's equations are formulated from the relativistic Schrödinger equation, and as we have seen the LG equations were the result of reasoning about the phase transition in fluids in relation to the free energy.

Therefore, the LG model would be described as a phenomenological model because it consists of: (1) Mathematical equations similar in form to a previous mathematical form. (2) A set of identifications that would relate parameters in the mathematical form, like α and β, with properties in superconductivity like coherence length and the penetration depth. Another important example is that of identifying the order parameter with the pseudo wave function. (3) A story that gives a description of the environmental set-up of the phenomenon of superconductivity; relates the different properties of superconductivity and displays how the mathematical forms might represent these properties. Furthermore, the LG model had been constructed by departing from the phenomenon.

The BCS theory

In 1957, the BCS theory was proposed taking into consideration the empirical facts stated earlier (note 7) and aiming to build a microscopic theory of superconductivity. While LG started from the experimental facts and tried to see how they could identify some data-outcome plot with Gordon's theoretical equation, the BCS theory was derived from quantum field theory. The belief that quantum field theory could establish a basis for understanding the properties of superconductivity came into place after an accumulation of microscopic models accounting for any aspect of superconductivity. Hence, the standpoint for BCS was a theoretical one, whereas for LG it was an experimental one.

The BCS derivation is long and complicated. It depends on a great number of theoretical tools to fulfil the constraints of the phenomenon. These tools are as follows.

(1) Previous physical theories: quantum field theory, electrodynamics, diamagnetism, Maxwell's equations and thermodynamics. These theories are what give legitimacy to the mathematical forms constructed by BCS to account for superconductivity. If BCS fail to provide a good and tidy derivation, using these accepted theories their theory will not qualify as a fundamental one. Though this may not be the only criterion for being a fundamental theory, it is a necessary one.

(2) Mathematical tools: the theory uses the quantum field theoretical technique of second quantization, perturbation theory, and different types of special approximations like Hartree–Fock-type. These abstract tools can help in investigating the existing facts so that they can be seen as similar to some part of an existing fundamental theory.

(3) A story: Here also we find a story is important to connect the previous tools with each other, and to connect

the mathematical parts of the theory to the natural phenomena. A metaphor here might be a good aid: while the story that the phenomenological model presents is strongly associated with the empirical findings, the story which the fundamental theory gives is like a piece of art (a sculpture or a painting) which the artist meant to be realist but, usually, turns out to be surrealist or even abstract. The point here is that the story given by the fundamental theory would use accepted theoretical models that might be justified only if we accept the underlying theoretical concepts carried on from previous fundamental theories. As we will see, in the case of the BCS theory, it depended on accepting ideas such as Fermi surface, the Cooper pair, electron–phonon interaction and Fermi sea. These theoretical models are all justified only if we accept quantum field theory as a point of departure.

Although generally physically motivated, identification of the mathematical terms with different sorts of real entities in the superconducting phenomenon does not usually tend to give a good description of the phenomenon, on the theoretical level. This point is important because such identification gives the mathematical models and tools a physics body. In our case, BCS runs the following story.

The BCS theory depends on a group of theoretical models that can associate superconductivity with quantum field theory. In 1955, Cooper suggested a way to understand the fact that the charge quanta in superconductivity are 2*e*. He suggested that the electrons in the superconducting state occur in correlated pairs (Cooper-pairs) – a theoretical model – that have the same quantum state. The importance of this point is obvious once we recall that electrons are fermions, which means they are unlikely to coexist in the same quantum state. But we can model a pair of electrons as a quasi-bosons. These pairs of electrons can be created through the electron–phonon *interaction* – a theoretical assumption – which considers that electrons interact with a lattice giving phonons. This idea was supported when the isotopic effect was observed – materials in nature are a combination of many isotopes, and the isotopic effect is the fact that the transition temperature depends on the isotopic nuclear mass. The idea that electron–phonon interactions are 'primarily responsible for superconductivity' seems reasonable, because it indicates that the vibrational motion of heavy nuclei plays an essential role in the formation of pairs of electrons. Let us remember that the relation between thermal vibration and conductivity was an established fact by that time.

The BCS theory also used the two-fluid model – a phenomenological model – which assumes that we can imagine the superconducting material as if it consists of two kinds of overlapping fluids, one of which is responsible for the normal state and the other for the superconducting state. This model can also help, in addition to the Cooper pairs, in understanding the use of another theoretical model, that of *Fermi surface*, which is an imaginary surface in *k*-space (spin-vector space) that separates the occupied energy levels from unoccupied energy levels and will define the first empty level.

We can see that BCS used some important assumptions (which I have underlined). These assumptions cannot be neglected in the theory; otherwise what will be left are merely the mathematical tools and bits and pieces from previous fundamental theories. However, right now all of these assumptions are being challenged.

It is important for any theory of superconductivity to derive the relation between the current density and both the potential and the momentum, because out of these two equations all the other known mathematical descriptions of the properties of the superconductors can be derived. For this derivation, using a quantum field theoretical framework, the BCS theory needs to employ all the mentioned models and tools from fundamental theories.

They start by suggesting a Hamiltonian for the electrons in the superconducting state. Then they add the isotopic mass, M_s, and its relation to the phonon–electron interaction, to see its effect on the non-diagonalized terms in the Hamiltonian. Then the diagonalized part is renormalized using 'Bloch energies'. Introducing the idea of the Fermi surface will allow them to use Fermi–Dirac statistics on matrix elements. The story goes on using annihilation and creation operators, then Hartree–Fock-like approximations, etc. They arrived in the end at a derivation for a special kind of wave function which, by defining the correct Hamiltonian and accepting a certain gauge where $\nabla\cdot\boldsymbol{A} = 0$, gives us a derivation for the paramagnetic and diamagnetic current densities:

$$\boldsymbol{j} = \boldsymbol{j}_p + \boldsymbol{j}_d,$$

$$\boldsymbol{j}_p(r) = \left\{ \begin{array}{l} -\dfrac{e^2\hbar^2(2\pi)^{3/2}}{2m^2c\Omega^2}\displaystyle\sum_{i\neq 0}\sum_{k,q,\sigma}\sum_{k',q',\sigma'}(2\boldsymbol{k}+\boldsymbol{q})k' \bullet \\ \left[\begin{array}{l} a(\boldsymbol{q}')e^{-\mathrm{i}q\cdot r}(\psi_0(T) \mid c^*_{k'+q',\sigma'}c_{k',\sigma'} \mid \\ \psi_i(t)) \times (\psi_i(T) \mid c^*_{k+q,\sigma}c_{k,\sigma} \mid \psi_0(T))\dfrac{1}{W_0 - W_i} \\ + \text{complex} - \text{conjugate} \end{array} \right] \end{array} \right.,$$

and

$$\boldsymbol{j}_d(\mathbf{r}) = -\left(\frac{ne^2}{mc}\right)\boldsymbol{A}(\boldsymbol{r}).$$

This, of course, can be accepted as a straightforward derivation from fundamental theory, namely quantum field theory.

It should be stated at this point that both the BCS and LG equations can account for the most important properties of superconductors, such as: (i) The penetration depth, i.e. the depth that a magnetic field can penetrate in a superconducting sample; the depth where the sample does not exhibit Meissner effect. (ii) The coherence length, which is ‘a measure of the distance within which the superconducting electron concentration cannot change drastically in a spatially-varying magnetic field’[16]. (iii) The energy gap. In the case of the BCS theory, it needs a series of approximations to account for each of these three properties.

Hence, the BCS theory fulfils the criterion suggested earlier in the article. It is consistent with and derivable from previous fundamental theory (quantum field theory), gives a coherent story about type-one superconductors, and can explain existing empirical findings.

Discussion

The previous sections showed that the LG model was built using a bottom-up approach, while the BCS theory was built using a top-bottom approach. Hence, the standpoint of the LG model was experimental evidence and that of the BCS model was quantum field theory. Let me discuss the differences between these two approaches. Bardeen[11] stated, way back in 1956, that:

> ‘Anything approaching a rigorous deduction of superconductivity from the basic equations of quantum theory is a truly formidable task. The energy difference between normal and superconducting phases at absolute zero is only of the order of 10^{-8} eV per atom. This is far smaller than errors involved in the most exacting calculations of the energy of either phase. One must neglect terms or make approximations which introduce errors which are many orders of magnitude larger than the small energy difference one is looking for. One can only hope to isolate the physically significant factors which distinguish the two phases. For this, considerable reliance must be placed on experimental findings and the inductive approach.’

So Bardeen, with whom Cooper and Schrieffer put forward the BCS theory of superconductivity, himself admits that any theory that departs from the quantum theory would need to ‘neglect terms or make approximations which introduce errors which are many orders of magnitude larger than’ the quantities one is looking for. The BCS theory needs exactly these approximations to account for practical situations.

On the contrary, the LG model relays on ‘experimental findings and the inductive approach’ and is able to present a mathematical structure that can be consistent with a representation of the phenomenon, trying to relate different bits and pieces from the shattered information provided through years of experimentation.

The BCS theory was accepted for a long time as the fundamental theory of superconductivity. There were different factors that contributed toward calling it ‘fundamental’, the most one being its use of quantum field theoretical grounds, i.e. a microscopic base for understanding a macroscopic phenomenon. The LG model also depended partly on microscopic factors – the Gordon’s formula – and LG employed their knowledge about fundamental theories to construct their model; yet nobody considered their model as fundamental. That was because their derivation did not give a clear reason for taking the order parameter to be a wave function; and because their derivation was not considered as a straightforward one from a previous fundamental theory.

Interestingly, the LG model has proved to be capable of adapting to new properties of superconductivity, whereas the BCS model failed to be modified to account for these new discoveries, as we shall see next. In this sense, LG can give an example of the way phenomenological models can prove more fruitful than fundamental theories.

Nevertheless, why did the BCS theory, in spite of all its success, fail to maintain its position as the fundamental theory of superconductivity? Up to a certain point, the BCS theory is reliable in giving an understanding of superconductivity. This is especially so if we are dealing with type-one superconductors. The BCS theory also managed, using further assumptions, to account for type-two superconductors. However, other kinds of superconductors, especially high-temperature superconductors which were discovered in 1986 by Bednorz and Müller, prove more problematic. It is important to mention here that in all the interpretations of the BCS theory concerning the critical temperature, the most optimistic one suggests 30 K to be the highest possible critical temperature. Now we have superconductors with (125 K) T_c (note 8). So the BCS theory cannot be seen as valid for all kinds of superconductivity.

In a discussion between Anderson and Schrieffer[19] on the difficulties facing a theory for high-temperature superconductivity, Anderson mentions: ‘I think few people realize that we now know of at least six different classes of electron superconductors, and two other BCS fluids as well. Out of these only one obeys the so-called conventional theory – that is, BCS with phonons that fit unmodified versions of Eliashberg’s equations’.

Anderson continues by stating that it is ‘crazy’ to think that the new high-temperature superconductors can fit the BCS theory, since even most of the simpler ones do not. He states that: ‘Back in the 1960s we may have created the abomination, a theory that has become “nonfalsifiable” in the Popperian sense in that people insist on inventing more and more ingenious ways to make it fit any anomaly!’.

In fact, that was quite right on the theoretical level; even a great physicist like Pippard mentioned in 1964 about the success of the BCS theory: ‘This success is so remarkable that I almost believe you would forgive me if I were to say there now remain no problems in superconductivity’[18]. Nevertheless, most physicists were reluctant to continue using the BCS techniques, especially after they found that the LG model could give them the same predictions with simpler mathematics. A survey of the textbooks on superconductivity can tell us about the role of the BCS theory. One of the most widely read textbooks was Michael Tinkham's *Introduction to Superconductivity*[20], where he writes: 'The emphasis is on the rich array of phenomena and how they may be understood in the simplest possible way. Consequently, the use of thermal Green Functions has been completely avoided, despite their fashionability and undeniable power in the hands of skilled theorists. Rather the power of phenomenological theory in giving insight is emphasized, and microscopic theory is often narrowly directed to the task of computing the coefficients in phenomenological equations'.

The BCS theory, as already stated, cannot be accepted as a genuine theory unless certain assumptions are associated with it. The new theoretical work is questioning each of these assumptions. In addition, the BCS theory (note 9) does not speak about important factors in high-temperature superconductivity, like the chemical structure of the materials and their normal state. Another reason for not accepting the BCS theory is its disagreement with new experimental outcomes: the highest critical temperature predicted by BCS is 30 K; the value of the energy gap that the BCS accepts is less than 3.5 kT_c, while the new superconductors exceed that limit to twice the value, etc.

Hence, while the fundamental theory of superconductivity failed to accommodate high-temperature superconductors and other kinds of superconductors, the LG phenomenological model of superconductivity proved to be more fruitful in representing all kinds of superconductors. The main difference between the BCS theory and the LGs model was the point of departure. Both of them used tools from previous fundamental theories. Both of them need a story to relate the mathematics with the properties of the phenomenon. Moreover, both of them hold a theoretical explanation. However, LG departs from the experimental level, while the BCS theory departs from the quantum field theory.

This important difference plays a major role in the type of story associated with the theoretical explanation. We saw that the BCS story needs certain assumptions to be consistent with the quantum field theory and to be able to derive the needed mathematical form. As shown above, all these assumptions are being challenged. The LG story depends on a well-tested set of assumptions, related to the empirical findings only. This gives the model the advantage of being attached to experimental evidence.

The association of the phenomenological model with experimental evidence, and the liberty which the model provides to some parameters to be measured experimentally, gives it a better stand toward representing new kinds of superconductors. Hence, the phenomenological model proved to be more fruitful than the fundamental theory of superconductivity.

A side issue: Is there a fundamental theory of superconductivity?

The striking thing about superconductivity is that it is a phenomenon that has two distinguishing properties, zero-resistance and Meissner effect (or magnetic vertex penetration), but these two do not have the same known origin. There are, as already mentioned, six known types of superconductors and each of them has different normal-state properties. The chemical properties of some are complicated and give rise to many contradictory results. Theory has so far failed to give a single generalized account for these different kinds of superconductivity. Many factors have been investigated in an attempt to account for superconductivity, but until now all of these factors appear to have experimental evidence against them[21].

It is important for any fundamental theory not to contradict any of the experimental observations that cannot be accepted as exceptions. Anderson[21] urged this kind of position in addressing the BCS assumption that all superconductors are Fermi liquid-type materials: 'Here I must appeal to a point of logic. The common response, when one makes a firm statement that none of these materials are Fermi liquids because of one or another observation, is to say that the observation encounters exceptions among these many materials. However, that is not the point: if they are all at the same fixed point – and they clearly are – it will be non-Fermi liquid for all if it is not for any one: it is necessary only to prove the negative in one instance. Exceptions are logically irrelevant'.

There is no fundamental theory for superconductivity. Physicists in the field still use the generalized LG phenomenological theory of superconductivity. This theory can equip physicists with effective mathematical techniques to predict the behaviour of a superconducting material or design superconducting devices. The major factor that makes the phenomenological models and theories so powerful is the fact that they are a first-level abstraction, departing from the experimental level.

The experimental observations now seem to indicate that it is highly probable that we will not be able to arrive at a fundamental theory of superconductivity that starts from existing fundamental theories. This is because the essential assumptions for the candidate theories, for such starting point, have proven to be in contradiction with experiments involving any of superconductors. That leads us, if we want to continue to search for fundamental theories, to one of two options.

The first option is to consider that the candidate theories are no good for superconductivity, but some other theory will emerge that can account for all the aspects of superconductivity. Of course, such a point of view does not tell us a lot, because fundamental theories should be, by definition, compatible with previous fundamental theories in the same domain of applicability. Einstein's theory, for example, should conflict with Newton's theory in the domain of low velocities and small masses. Hence, the predictions of the new theory should not be in contradiction with the well-confirmed predictions of the previous theories. So this option will require a whole new theoretical approach, not just for superconductivity, but for other domains as well.

The other option is to consider that there is more than one fundamental theory for superconductivity: one each for conventional superconductors, high-temperature superconductors, organic superconductors, etc. This option might work, though it would end up by contradicting the unification assumption of fundamental theories, i.e. fundamental theories are compatible with each other and there will be a way in which all these theories can be unified in a theory of everything.

None of these two options is necessary. If we accept that phenomenological models are representative of nature, then our theories will eventually be merely tools to help us in constructing new theoretical tools and new phenomenological models. Science and scientists will have more freedom by doing that, and this will help them to go beyond the theoretical limitations. After all, if all the physicists had accepted the BCS theory, we would have never been able to discover high-temperature superconductors.

Notes

1. In a previous paper with Nancy Cartwright and Mauricio Suárez[2], we argued that theories are tools to construct phenomenological models. The idea presented here is a step further.
2. There are, however, some cases where another practice can be found in physics: a theory predicts, due to certain hypothesis, the occurrence of a phenomenon under such and such circumstances. The theory also provides a theoretical model to describe the idealized situation predicted by it. If the technological developments can permit applying the prediction, a set of experiments would be conducted to test the hypothesis. During the course of testing, a procedure of approximations and modifications is applied on the theoretical model and most of the time we end up having a new model which has many elements that are not relevant to the original theory. I will call such a model also a phenomenological model. It is important to notice here that the new model, although the phenomenon was predicted by the theory, cannot – at most times – be deduced from the postulates of the theory because of introducing new ‘irrelevant’ factors to the theory and is not deducible from it[3].
3. As Margaret Morrison claims in her reply to Ian Hacking[4].
4. For a detailed discussion of London and London model see Suarez[8].
5. Second-order transition means that g_4 is positive in contrast with the first-order transition, where g_4 is negative and we cannot neglect the rest of the terms.
6. This kind of associating a form from relativistic Schrödinger equation with a derived form from another field is form correspondence. For details about form correspondence and its role in physics see Shomar[1].
7. The BCS paper starts as follows: ‘The main facts which a theory of superconductivity must explain are (1) a second-order phase transition at the critical temperature, T_c, (2) an electronic specific heat varying as $\exp(-T_0/T)$ near $T = 0$ K and other evidence for an energy gap for individual particle-like excitations, (3) the Meissner–Ochsenfeld effect ($B = 0$), (4) effects associated with infinite conductivity ($E = 0$), and (5) the dependence of T_c on isotopic mass, $T_c vM = \text{const}$’[15].
8. For a historical account of the developments in the field of superconductivity see Schechter[17] and Vidali[18].
9. The BCS theory was accepted as a fundamental theory because of its ability, according to the known features of superconductivity at the time, to represent a coherent explanation of the phenomenon. Nonetheless, it failed to account for many elements and was not able to explain new features of superconductivity. In this regard, although the LG model does not have a rigorous derivational origin, it was open to changes and was able to account for the newly discovered high-temperature superconductors.

1. Shomar, T., Phenomenological realism, superconductivity and quantum mechanics, Ph D thesis, London School of Economics, London, 1998.
2. Cartwright, N., Shomar, T. and Suárez, M., The tool-box of science. In *Theories and Models in Scientific Processes* (eds Herfel, W. *et al.*), 1995, vol. 44, pp. 137–149.
3. Shomar, T., Modelling in applied physics: The case of polymers. *Dirasat, Pure Science*, 2006, **33**, 241–250.
4. Morrison, M., *Unifying Theories: Physical Theories and Mathematical Structures*, Cambridge University Press, Cambridge (in press).
5. Grant, I. S. and Philips, W. R., *Electromagnetism*, John Wiley, New York, 1982, p. 73.
6. Burns, G., *High-Temperature Superconductivity: An Introduction*, Academic Press, San Diego, 1992, p. 10.
7. London, F. and London, H., The electromagnetic equations of the supra-conductor. *Proc. R. Soc. London, Ser. A*, 1935, **149**, 71.
8. Suarez, M., The role of models in application of scientific theories: Epistemological implications. In *Modals as Mediators* (eds Morrison and Morgan, M.), Cambridge University Press, Cambridge, 1999.
9. Ginzburg, V. L., Some remarks concerning the macroscopic theory of superconductivity. *Sov. Phys. JETP*, 1956, vol. 2, pp. 589–600.
10. Sproull, R. and Phillips, W., *Modern Physics: The Quantum Physics of Atoms, Solids and Nuclei*, John Wiley, New York, 1976.
11. Bardeen, J., Theory of superconductivity. *Encyclopaedia of Physics*, Springer-Verlag, Berlin, 1956.
12. Schrieffer, J. R., *Theory of Superconductivity*, W.A. Benjamin Inc., New York, 1964, p. 19.
13. Tilley, D. and Tilley, J., *Superfluidity and Superconductivity*, Adam Hilger Ltd, Bristol, 1986.
14. De Gennes, P. G. and Pincus, P. A., *Superconductivity of Metals and Alloys*, W. A. Benjamin Inc, New York, 1966, p. 176.
15. Bardeen, J., Cooper, L. N. and Schrieffer, J. R., Theory of superconductivity. *Phys. Rev.*, 1957, **108**.
16. Kittel, C., *Introduction to Solid State Physics*, John Wiley, New York, 1986, p. 336.
17. Schechter, B., *The Path of No Resistance*, Simon Schuster, New York, 1989.
18. Vidali, G., *Superconductivity the Next Revolution?*, Cambridge University Press, Cambridge, 1993.
19. Anderson, P. and Schrieffer, R., A discussion of superconductivity. *Physics Today*, June 1991, p. 54.
20. Tinkham, M., *Introduction to Superconductivity*, McGraw-Hill, New York, 1975.
21. Anderson, P., *Science*, 12 June 1992, **256**.

Received 9 October 2007; revised accepted 28 March 2008